Charge transport through weakly open one-dimensional quantum wires

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We consider resonant transmission through a gated finite-length quantum wire connected to leads via finite-transparency junctions, such that the escape time is much smaller than the energy relaxation time in the wire. The coherent electron transport is strongly modified by the Coulomb interaction. The low-temperature current-voltage (*IV*) curves show steplike dependence on the bias voltage determined by the distance between the quantum levels inside the conductor, the pattern being dependent on the ratio between the charging energy and level spacing. If the system is tuned close to the resonance condition by the gate voltage, the low-voltage *IV* curve is ohmic. At large Coulomb energy and low temperatures, the conductance is temperature independent for any relationship between temperature, level spacing, and coupling between the wire and the leads.

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I. INTRODUCTION

Quantization of conductance in ballistic one-dimensional (1D) channels or islands[1] is one of the central issues of current mesoscopic physics (see Ref. 2 for a review). In small-size islands, the Coulomb blockade suppresses conductivity at low temperatures and small applied voltages.[3] The Coulomb blockade can be controlled by varying the charge of the island between two tunnel barriers by the potential at the gate electrode. The classical theory of the Coulomb-blockade oscillations was developed in Ref. 4, while the role of the discreteness of the spectrum was addressed in Refs. 5 and 6 and several subsequent papers. In these models the island was assumed to be almost isolated from both source and drain so that the number of particles on it could be considered a conserved quantity. This assumption, in general, does not hold if the transparency of the contacts between the island and the leads is finite.[7] Typical examples of such systems are quantum dots.[8]

Transport through quantum conductors (QCs) essentially coupled to the leads has not been fully investigated so far, though it offers excellent opportunities for studying the interplay between quantum and classical properties of QC. In this work, we investigate transport through a relatively short single-mode ballistic QC assuming a simplest model where charging effects are important, whereas the Luttinger-liquid behavior[9,10] is still not pronounced (for estimates see, e.g., Refs. 11 and 12). The conductor is connected to two leads via identical contacts with a finite transition amplitude $\mathcal{T}$. Its length d is shorter that the electron mean-free path, the transport mechanism being the *resonant transmission*, which we distinguish from elastic cotunneling.[13] This mechanism is relevant to recent experiments on carbon nanotubes.[14,15]

At $\mathcal{T}\to 0$, the QC holds a fixed number of particles, N. If a particle with energy E tunnels between a lead and the state with energy ϵ_p in the QC, the charging energy of the QC couples E and ϵ_p through[6]

$$E = \epsilon_p + 2E_C(N - N_0) \pm E_C, \tag{1}$$

where $\pm$ stands for adding/removing a particle, $E_C = e^2/2C$ is the Coulomb energy of the QC, C is the QC capacitance, and eN_0 is the gate-induced charge density. At finite $\mathcal{T}$, the system forms a double-barrier resonant tunneling structure, its properties being determined by the resonant states. The distance between these resonant levels is $\Delta E = \pi\hbar v_F/d$ (where v_F is the electron velocity) while their width is $\Gamma = |\mathcal{T}|^2\Delta E$. The occupation probability of the modes is determined by the competition between the relaxation processes in the QC and in the leads, as well as by escape probability from the QC into the leads.

Relaxation in the QC is determined by interplay between the charge relaxation time τ_Q, which includes the flight time and the RC time,[16,17] the escape time $\hbar/\Gamma$, and the energy relaxation time τ_ϵ due to inelastic electron-phonon and electron-electron processes in the QC. We will show later that, in our situation, the RC time, $\tau_C = RC$, is much shorter than the inelastic relaxation time τ_ϵ. In the ballistic limit, the flight time d/v_F is also shorter than τ_ϵ. If the contact transmission is not very low, such that $\hbar/\Gamma \ll \tau_\epsilon$, the energy relaxation time τ_ϵ appears to be the longest time scale. As a result, a homogeneous distribution of charge and potential in the conductor establishes quickly while the level occupation relaxes much slower, leading to pronounced effects in the conductance. In this case the distribution function inside the QC is determined by the coupling to the leads; it can strongly deviate from the thermal Fermi distribution.

We will assume here that the resonance width Γ is small compared to the interlevel spacing, but is still larger than the inelastic relaxation rate in the QC,

$$\hbar/\tau_\epsilon \ll \Gamma \ll \Delta E. \tag{2}$$

Requirement (2) is opposite to the limit $\hbar/\Gamma \gg \tau_\epsilon$ considered previously in many publications.[6,18–22] In that case the occupation probability is given by the equilibrium Fermi distribution with the lattice temperature, the chemical potential

being determined by the (conserved) number of particles.

Under the condition of Eq. (2) considered below the QC cannot be treated as an isolated quantum dot with the vanishingly weak coupling to leads,[23] and therefore, N is not a good quantum number but rather an average value determined by the interaction with the leads. To allow for this we employ the mean-field approach which leads to modification of Eq. (1). We show that at finite $\mathcal{T}$ the excitation spectrum changes significantly. At large Coulomb energy, $E_C > \Delta E$, and low temperatures, $T \ll E_C$, energy exhibits a sharp step as a function of the internal momentum in the QC. This step defines the internal Fermi level. The width of the step is determined by the width of the resonant level. As a result, zero-voltage conductance becomes temperature-independent in a rather broad temperature domain $T \ll E_C$ irrespective of the relationship between T, ΔE, and Γ. In a carbon nanotube the ratio $E_C/\Delta E = e^2 d/2\pi\hbar v_F C$ can reach values $e^2/2\pi\hbar v_F$ for the minimal capacitance of the tube $C \sim d$. For typical $v_F = 0.8 \times 10^8$ cm/s (see, e.g., Ref. 15) this ratio is ≈ 0.46, i.e., is of the order of unity. Therefore, in order to achieve better understanding of the experimental situation and having in mind more general applications, both limits of large and small ratio $E_C/\Delta E$ have to be studied.

II. MODEL

A. Hamiltonian

We specify the charging Hamiltonian as

$$\hat{H}_C = E_C \left[\sum_\alpha \int_{\mathcal{V}} \hat{\psi}^\dagger_\alpha(\mathbf{r}) \hat{\psi}_\alpha(\mathbf{r}) d\mathcal{V} - N_0 \right]^2, \tag{3}$$

where $\mathcal{V}$ is volume of the QC and α is the spin index. The spin dependence is due to the level filling that controls the charge on the QC. This is the simplest and the most widely used form of the charging Hamiltonian; note that it ignores both the Luttinger-liquid (LL) effects and inhomogeneities in the charge distribution.[12,24]

The LL parameter g that characterizes the electron-electron Coulomb interaction strength is coupled to the ratio $E_C/\Delta E \sim e^2/2\pi\hbar v_F$ by $e^2/2\pi\hbar v_F \sim g^{-2} - 1$. For strong Coulomb interaction one has $g \ll 1$. As has been shown in Ref. 12, the LL behavior in a one-dimensional conductor of length d connected to the leads becomes important only when the bias voltages are quite high, $eV/\Delta E \gtrsim g^{-1}$. For lower voltages the LL effects are small. We see that for strong Coulomb interaction, $g \ll 1$, the region of Fermi-liquid behavior extends up to moderate voltages $eV \gtrsim \Delta E$. The LL corrections also vanish in the limit of small interaction, $e^2/2\pi\hbar v_F \ll 1$. Therefore, our model that avoids the LL effects is reasonable up to the voltages of the order of several interlevel distances.

The use of an effective capacitance in our model is justified because the uniform charge distribution in the QC establishes quickly. Indeed, in what follows we are interested in the resonance transmission. Near a resonance, the resistance of the double-barrier structure is of the order of the resistance quantum $R \sim G_0^{-1} = 2\pi\hbar/e^2$. Therefore, the RC time is

$$\tau_C \sim C\hbar/e^2 = \hbar/2E_C \sim (\Delta E/E_C)(d/v_F).$$

The flight time d/v_F is much shorter than the inelastic time in the ballistic limit. Therefore, there exists a wide window extending from large to small magnitudes of the interaction parameter $E_C/\Delta E$ for which $\tau_Q \ll \tau_\epsilon$.

The mean-field approximation reduces Eq. (3) to the equation for the retarded and advanced Green's functions $G^{R(A)}_\epsilon$

$$G_0^{-1}(\epsilon, \mathbf{r}_1) G_\epsilon(\mathbf{r}_1, \mathbf{r}_2) + \int W(\mathbf{r}_1, \mathbf{r}) G_\epsilon(\mathbf{r}, \mathbf{r}_2) d\mathcal{V} = \delta(\mathbf{r}_1 - \mathbf{r}_2) \tag{4}$$

containing the effective nonlocal interaction in the QC,

$$W(\mathbf{r}_1, \mathbf{r}) = 2E_C h(\mathbf{r}_1) h(\mathbf{r}) \int \frac{d\epsilon}{4\pi i} G^K_\epsilon(\mathbf{r}_1, \mathbf{r}), \tag{5}$$

which is expressed through the Keldysh function, G^K. The factor $h(\mathbf{r}) = 1$ if $\mathbf{r}$ belongs to the QC, and zero otherwise,

$$G_0^{-1}(\epsilon, \mathbf{r}_1) \equiv -\epsilon + \hbar^2 \hat{\mathbf{p}}_1^2/2m - \mu + U_C h(\mathbf{r}_1) + V_b(\mathbf{r}_1),$$

$\hat{\mathbf{p}}_1 \equiv -i\partial/\partial \mathbf{r}_1$. The energy $U_C = e^2(N - N_0)/C - e\varphi$ is produced by the charge on the QC and the potential φ produced by the gate electrode (N_0 is the number of electrons for zero gate voltage); μ is the chemical potential; $V_b(\mathbf{r}_1)$ is the potential of the barriers at the contacts.

For methodical purposes, let us start with the example of very long QC under zero-bias voltage assuming $\Delta E \ll E_C$. Since E_C is coordinate independent one can consider the wave functions as plane waves, and use momentum representation. Under the condition of Eq. (2) the distribution function in the island is determined by the leads. We have in Eq. (5)

$$G^K_\epsilon(\mathbf{r}_1, \mathbf{r}) = [G^R_\epsilon(\mathbf{r}_1, \mathbf{r}) - G^A_\epsilon(\mathbf{r}_1, \mathbf{r})] \tanh \frac{\epsilon}{2T}$$

$$= 2\pi i \int \frac{dp}{2\pi} e^{ip(r_1 - r)} \delta(\varepsilon_p - \epsilon) \tanh \frac{\epsilon}{2T},$$

where ε_p is the energy spectrum. If the island is long we can extend the coordinate integration from $-\infty$ to $+\infty$ and find from Eq. (4)

$$\left(\xi_p + U_C - \epsilon + E_C \tanh \frac{\varepsilon_p}{2T} \right) G^R_\epsilon(p) = 1.$$

The Green's function has the pole at $\epsilon = \varepsilon_p$, where

$$\varepsilon_p = \xi_p + U_C + E_C \tanh \frac{\varepsilon_p}{2T}, \quad \xi_p = \frac{\hbar^2 p^2}{2m} - E_F. \tag{6}$$

ε_p is measured from the Fermi level $E_F = \hbar^2 p_F^2/2m$ in the leads. At $\varepsilon_p \gg T$ this coincides with Eq. (1). For $\varepsilon_p \to 0$ one has $\varepsilon_p = (\xi_p + U_C)/(1 - E_C/2T)$. The slope of the ε_p vs ξ_p dependence is negative for $E_C > 2T$, and the function ε_p becomes multivalued [see Fig. 1(a)]. As we will see later, ε_p in fact experiences an abrupt jump shown in Fig. 1(a) by the vertical line. This infinite slope is a consequence of the chosen approximation of an infinitely long QC with $\Delta E \to 0$. In

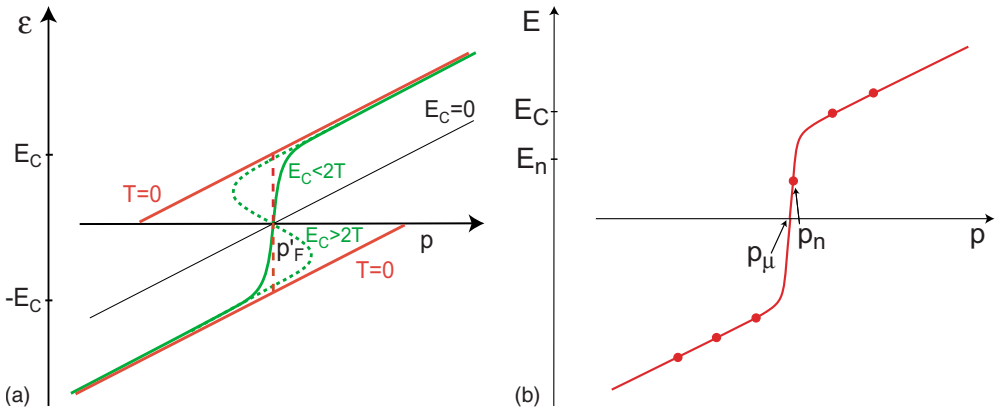

FIG. 1. (Color online) (a) Energy spectrum of an infinitely long conductor near the Fermi energy. Finite T (green lines): The slope at $E=0$ is positive for $E_C<2T$ (full line) and negative for $2T<E_C$ (dotted line). For $T=0$ (red lines) the Fermi momentum determines the jump in the spectrum; $p_F'=p_F-U_C/\hbar v_F$ (dashed vertical line). Straight (black) line: zero Coulomb energy. (b) Energy spectrum for finite-transparency contacts. The dots illustrate positions of the states.

a finite QC, the jump acquires a finite width determined by the transparency of the contacts.

B. Method

To study this problem in more detail it is convenient to expand Green's functions over the orthonormal set of functions $u_n(\mathbf{r})$,

$$G_\epsilon^{R(A)}(\mathbf{r}_1,\mathbf{r}_2)=\sum_n \frac{u_n(\mathbf{r}_1)u_n^*(\mathbf{r}_2)}{E_n-\epsilon\mp i\delta},$$

where $u_n(\mathbf{r})$ satisfy the Schrödinger equation

$$\left[\frac{\hbar^2\hat{\mathbf{p}}^2}{2m}-\mu+U_C h(\mathbf{r})+V_b\right]u_n+E_C\sum_m u_m N_{mn}=E_n u_n. \quad (7)$$

Here $N_{mn}=\int_{\mathcal{V}} d\mathcal{V} u_m^*(\mathbf{r})u_n(\mathbf{r})f(E_m)$; $f(E_m)\equiv 1-2n(E_m)$, where $n(E_m)$ is the occupation probability of the mth state. The diagonal element is proportional to the average charge eN_{nn} in the state n. While obtaining Eq. (7) we used the Keldysh function in the form

$$G_\epsilon^K(\mathbf{r}_1,\mathbf{r}_2)=2\pi i\sum_n \delta(\epsilon-E_n)u_n(\mathbf{r}_1)u_n^*(\mathbf{r}_2)f(E_n).$$

Since main contributions to the transport come from the quasiresonant tunneling states it is convenient to expand the functions u_n over the scattering states, $v_p(x)$, satisfying the 1D equation

$$\left[\frac{\hbar^2\hat{p}_x^2}{2m}-\mu+V_b(x)\right]v_p(x)=\xi_p v_p(x), \quad (8)$$

$V_b(x)=V_0\Sigma_\pm\delta(x\pm d/2)$. The wave functions can be chosen as incident waves on the left, v_p, and incident waves on the right, $v_{\bar{p}}$. We assume a symmetric structure such that each barrier is characterized by the same plane-wave reflection $\mathcal{R}$ and transmission $\mathcal{T}$ amplitudes, $\mathcal{R}=|\mathcal{R}|e^{i\delta}$ and $\mathcal{T}=-|\mathcal{T}|e^{i\delta}$, where δ is the scattering phase; $|\mathcal{R}|^2+|\mathcal{T}|^2=1$.

The states with an incident particle on the left, $v_p(x)$, and on the right, $v_{\bar{p}}(x)$ are, respectively,

$$v_p(x)=\begin{cases} e^{ipx}+re^{-ipx}, & x<-d/2 \\ te^{ipx}, & x>d/2 \end{cases}, \quad (9)$$

$$v_{\bar{p}}(x)=\begin{cases} e^{-ipx}+re^{+ipx}, & x>d/2 \\ te^{-ipx}, & x<-d/2 \end{cases}. \quad (10)$$

Here r and t are the reflection and transmission amplitudes for the double-barrier structure. We will also use the even and odd functions

$$v_p^{\pm(0)}(x)=\frac{1}{\sqrt{2}}[v_p(x)\pm v_{\bar{p}}(x)].$$

For $|x|<d/2$ the functions are

$$v_p^{\pm(0)}(x)=\frac{\mathcal{T}e^{-ipd/2}}{\sqrt{2}}\left[\frac{e^{ipx}\pm e^{-ipx}}{e^{-ipd/2}\mp\mathcal{R}e^{ipd/2}}\right]. \quad (11)$$

The free functions satisfy

$$\int_{-\infty}^{\infty} v_p^{\pm(0)}(x)v_{p'}^{*\pm(0)}(x)dx=2\pi\delta(p-p'), \quad (12)$$

$$\int_0^{\infty} v_p^{\pm(0)}(x)v_p^{*\pm(0)}(x')\frac{dp}{2\pi}=\delta(x-x'). \quad (13)$$

The reflection and transmission coefficients correspond, by definition, to the plane waves,

$$r_p\pm t_p=e^{-ipd+i\delta}\frac{1\mp|\mathcal{R}|e^{-ipd-i\delta}}{e^{-ipd-i\delta}\mp|\mathcal{R}|}.$$

We now express the wave functions u_n through the free functions v_p,

$$u_n(x)=\int A_{np}v_p(x)\frac{dp}{2\pi}.$$

Since $u_n(x)$ are also orthonormal, we have

$$\sum_n A^{\pm}_{np} A^{*\pm}_{np'} = 2\pi\delta(p-p'), \quad \int \frac{dp}{2\pi} A^{\pm}_{np} A^{*\pm}_{n'p} = \delta_{n,n'}. \tag{14}$$

Using the orthogonality and completeness of both sets u_n and v_p, Eqs. (12)–(14), one can show that the states u_n with the different n are expanded in v_p with different p. This property, in particular, excludes the multivalued solutions of Eq. (6) shown in Fig. 1(a). Expanding Eq. (7) into wave functions $v_p(x)$ we obtain

$$A_{np}(\xi_p - E_n) + \int \frac{dp'}{2\pi} A_{np'}[U_C M_{pp'} + E_C Q_{pp'}] = 0, \tag{15}$$

where $\xi_p = \hbar^2 p^2/2m - E_F$ and

$$Q_{pp'} = \sum_m \int \frac{dp_1 dp_2}{(2\pi)^2} A_{mp_1} A^*_{mp_2} f_1(E_m) M_{pp_1} M_{p_2 p'}.$$

Here introduce the matrix elements

$$M_{pp'} \equiv \int_{-d/2}^{d/2} v^*_p(x) v_{p'}(x) dx,$$

which can be explicitly expressed through the transmission amplitude, $\mathcal{T}$, as

$$M^{\pm}_{pp'} = \frac{\mathcal{T}_{p'} \mathcal{T}^*_p e^{i(p-p')d/2} \Delta(p-p')}{(e^{-ip'd/2} \mp \mathcal{R} e^{ip'd/2})(e^{ipd/2} \mp \mathcal{R}^* e^{-ipd/2})} \tag{16}$$

with $\Delta(p) \equiv (2/p)\sin(pd/2)$. The upper (lower) signs refer to the even (odd) states in the QC. For $|\mathcal{T}| \ll 1$ there exist sharp resonances in the transmission when the momentum of electrons inside the QC is close to the resonant values corresponding to integer ratio $s \equiv (pd+\delta)/\pi$, $\delta \approx \pi$. The even and odd resonant states correspond to $s=2n+1$ and $s=2n$, respectively. Note that the factors M are nearly orthogonal,

$$\int M^{\pm}_{pp_1} M^{\pm}_{p_1 p'} \frac{dp_1}{2\pi} \approx 2\pi\delta(p-p'), \tag{17}$$

if both p and p' are close to the same resonance, $|p - q_n|, |p' - q_n| \ll d^{-1}$.

It is important that the true wave functions $u_n(r)$ do also have resonances inside the QC. The presence of resonances implies, in particular, that transport is controlled only by discrete resonant levels, which distinguishes the considered mechanism from the usual elastic cotunneling through a multilevel quantum dot.[13] At $E_C \gg \Delta E$ the corresponding internal resonance momenta of $u_n(r)$ lie far from the particle momenta in the leads $\hbar k = \sqrt{2m(E+E_F)}$ due to a large interaction energy in the QC. Nevertheless, each resonance state of $u_n(r)$ inside the QC is close to some resonance state of the plane wave $v_p(x)$ with a momentum $\hbar p$ that is, in general, distinct from $\hbar k$. Keeping this in mind, one can put $A_{np} \approx M^*_{q_n p} = M_{p q_n}$, where the continuous parameter q_n is the wave vector of a particle inside the QC close to nth resonance. This allows us to simplify Eq. (15) in the vicinity of resonances.

III. RESULTS AND DISCUSSION

A. Spectrum

We concentrate on the situation when the transparency satisfies inequalities (2). If there is no bias voltage, the distribution functions for particles coming from the left and right are the same, $f(E) = \tanh(E/2T)$. Using orthogonality condition (17) we obtain from Eq. (15)

$$E_n = \xi_{q_n} + U_C + E_C \int \frac{dp}{2\pi} f(E_p) M_{pp}. \tag{18}$$

Here $\xi_{q_n} = \hbar^2 q_n^2/2m - E_F$, the integration is performed over the vicinity of the nth resonance. Note that that in approximation (17) the off-diagonal terms N_{nm} $(m \neq n)$ present in Eq. (7) vanish. Equation (18) generalizes Eq. (6) with the replacements $\varepsilon_p \to E_m$, $\xi_p \to \xi_{q_n}$ and defines the value q_n that corresponds to the resonant transmission through the QC. The inelastic relaxation is the slowest intra-QC relaxation in the ballistic limit since the RC time is of the order of the flight time d/v_F for $E_C/\Delta E \sim 1$. Were the inelastic processes efficient, $\tau_\epsilon \ll \hbar/\Gamma$, the distribution would be determined by the states in the QC, $f(E_p) \equiv f(\xi_{q_n}) = \tanh(\xi_{q_n}/2T)$. The abrupt dependence shown in Fig. 1(a) would be then smeared by the finite temperature, and E_n as a function of q_n would acquire a finite width $\sim T$, similarly to the situation considered in Ref. 6. However, if Eq. (2) holds, and when temperature satisfies $T \ll E_C$, the spectrum $E_n(q_n)$ shows a jump over $2E_c$ at some value $q_n \equiv p_\mu$ such that all the levels with $q_n > p_\mu$ are empty, while the levels with $q_n < p_\mu$ are occupied. Therefore the level population strongly deviates from the Fermi distribution: $f(E_n) \approx \mathrm{sign}(q_n - p_\mu)$. With this approximation in the right-hand side (rhs) of Eq. (18) one finds

$$E_n = \hbar v_F (q_n - p_F) + U_C + E_C \Phi(p_n - p_\mu). \tag{19}$$

Here $\hbar p_F$ is the Fermi momentum in the leads; q_n is close to the resonance value $p_n \equiv \pi n/d$ and

$$\Phi(p_n - p_\mu) = (2/\pi)\arctan[2d(p_n - p_\mu)/|\mathcal{T}|^2]. \tag{20}$$

Equation (19) is one of the central results of this paper; it generalizes Eq. (1) for low transparency limited by Eq. (2). Equations (19) and (1) coincide far from the Fermi level. Energies E_n for the resonance momenta p_n are shown in Fig. 1(b). The width of this function is determined by Γ, rather than by the temperature.

B. Current

The Fermi momentum, p_μ, is related to the number of electrons inside the QC. We define the variation of the average particle number, δN, as a function of the bias and gate voltage. Using Eqs. (19) and (20) one can cast this variation in the form

$$\delta N = -\delta \sum_n \frac{1}{2} \Phi(p_n - p_\mu).$$

If the average number of electrons for zero gate voltage, N_0, is an integer, the chemical potential lies far from one of the resonances $|\mathcal{T}|^2/d \ll |p_\mu - p_n| \leq \pi n/2d$. Near the degeneracy

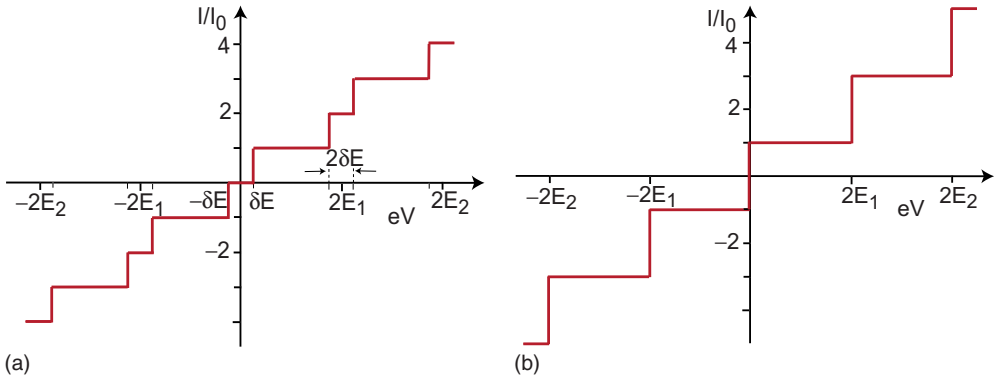

FIG. 2. (Color online) (a) Current steps for zero Coulomb energy and $T=0$. $E_n=\Delta En$ is the resonance energy, $\delta E=\hbar v\alpha'/d$ is the deviation from the resonance. (b) For exact resonance, $\delta E=0$, the steps are doubled.

point when $N-[N_0]=\pm 1/2$ ([…] is the integer part), the Fermi momentum is close to one of the resonances p_n. In general, the chemical potential and the number of particles are related by the equation $2(N-[N_0])+\Phi(p_n-p_\mu)=1$.

For a finite bias V, energies in the left and right leads are shifted by $\pm eV/2$. Following the same approximation as above, we have instead of Eq. (19)

$$E_n=\hbar v_F(q_n-p_F)+U_C+\frac{E_C}{2}\sum_{i=L(R)}\Phi(p_n-p_\mu^{(i)}).$$

Now the Fermi momenta of electrons coming from left or right electrode, $\hbar p_\mu^L$ and $\hbar p_\mu^R$, are not equal. Similarly to the equilibrium case, we get

$$\delta N=-\delta\sum_{n,i=L(R)}\frac{1}{4}\Phi(p_n-p_\mu^{(i)}).$$

Defining

$$j_p\equiv u_p^*(x)[\partial u_p(x)/\partial x]$$

the current becomes

$$I=-\frac{e\hbar}{m}\sum_{p,\alpha}[f^L(E_p)\mathrm{Im}\,j_p+f^R(E_{\bar{p}})\mathrm{Im}\,j_{\bar{p}}]. \quad (21)$$

The following analysis is different for small and large values of $E_C/(\Delta E)$. At $E_C\ll\Delta E$, the energy $E_n=\hbar v_F(q_n-p_\mu)$ is independent of the level population and thus is doubly degenerate in spin state. At relatively high temperatures, $\Gamma\ll T\ll\Delta E$, using Eq. (21), one obtains

$$I=\frac{I_0}{2}\sum_n\left[\tanh\frac{E_n^+}{2T}-\tanh\frac{E_n^-}{2T}\right],\quad I_0\equiv\frac{ev_F}{2d}|\mathcal{T}|^2. \quad (22)$$

$E_n^\pm\equiv E_n\pm eV/2$. Equation (22) generalizes the high-capacitance Landauer-Büttiker result (compare with Refs. 6 and 25) to bias voltages up to the values corresponding to several interlevel distances. The IV curves are shown in Fig. 2. The current exhibits steps at

$$eV=2(\Delta En\pm\delta E),$$

where $\delta E=(\hbar v_F/\pi)[p_\mu-\pi M/d]$ is the deviation from resonance between the zero-voltage Fermi energy in the leads and one of the levels, M being an integer.

For $T\ll\delta E$ and small voltages, $eV\leq\delta E$, the current is zero. For the resonance, $\delta E=0$, the current is given by the term with $n=M$ in the sum, $I=I_0\tanh(eV/4T)$. At $eV\ll T$ the conductance is ohmic, $G=eI_0/4T$.

For large E_C, the states within the interval $\delta p\sim|\mathcal{T}|^2/d$ near the lowest resonance level, i.e., those which contribute to the current, have energies $\sim E_C$. Therefore the distribution function $\mathrm{sign}(q_n-p_\mu)$ can be used up to temperatures $T\ll E_C$, regardless of the relation between T and Γ or ΔE. Using Eq. (21) we find

$$I=(I_0/4)\sum_n[\Phi(p_n-p_\mu^R)-\Phi(p_n-p_\mu^L)]. \quad (23)$$

The level positions are determined by the condition of minimal total energy which fixes the value of $N-N_0$. Thus the levels, as functions of the bias voltage, will cross the Fermi energy pairwise, one level from above while the other from below, keeping the number of electrons $N-N_0$ unchanged and lifting the Coulomb blockade in the bias voltage. If the Fermi level lies between the resonances, the first step of height I_0 in the current appears for $V=\Delta E/e$. The next steps appear when the bias voltage is increased by $\delta V=2\Delta En/e$. The heights of the current steps are the same as for low E_C: the extra factor $\frac{1}{2}$ in Eq. (23) is compensated by the pairwise level crossing.

If the system is close to the degeneracy point, $N-[N_0]=\pm 1/2$, when one of the values p_n is close to p_μ, even a small bias voltage is sufficient to produce current. Since $p_\mu^L-p_\mu^R=eV/\hbar v_F$ and $p_\mu^L\approx p_\mu^R\approx p_\mu$ we find from Eq. (23) for low voltage, $eV\ll\Gamma$,

$$G\equiv\frac{I}{V}=\frac{e^2}{2\pi\hbar}\frac{|\mathcal{T}|^4}{|\mathcal{T}|^4+4(p_n-p_\mu)^2d^2}. \quad (24)$$

In the resonance, $p_n=p_\mu$, the conductance, G, is the single-spin conductance quantum, $e^2/2\pi\hbar$. Equation (24) is applicable sufficiently close to a resonance, $4(p_n-p_\mu)^2d^2\lesssim|\mathcal{T}|^4$. Otherwise this contribution is small, being beyond the accuracy of the employed approximation; it has the same order of magnitude as that of cotunneling, which is not taken into account here.

For large charging energy $E_C \gg \Delta E$, the current, Eq. (23), and the conductance defined by Eq. (24) do not depend on temperature at $T \lesssim E_C$. This fact distinguishes our results, Eqs. (23) and (24), from the predictions of Refs. 6, 18, and 19, where the similar expressions were found only within the temperature domain $T \ll \Gamma$. The difference is due to the presence of the step in the energy spectrum caused by relatively strong coupling to the leads. Probing the onset of ohmic conductance at the degeneracy point allows one to monitor the effective number of electrons in the QC.

In conclusion, we have developed a mean-field description of the Coulomb effects in the "weakly open" 1D systems and analyzed both the excitation energy spectrum and the electric current. The *IV* curves show a steplike dependence on the bias voltage, the exact shape being determined by the ratio of the charging energy to the level spacing. At large charging energy and low temperatures $T \ll E_C$, the low-voltage ohmic conductance, Eq. (24), is temperature-independent irrespectively to the magnitude of the ratios $T/\Delta E$ and T/Γ. This is due to a nonequilibrium population of levels in the QC caused by strong coupling to the leads, which distinguishes our results from those obtained for weakly coupled conductors with an equilibrium population of the energy levels.

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